<u>Part 13</u>: Heat and Thermodynamics

College Physics (Openstax): Chapters 13, 14, & 15 Physics (Giancoli): Chapters 13, 14, & 15

Temperature Scales

- The <u>Fahrenheit</u> <u>Scale</u>, proposed by Daniel Fahrenheit in 1724 and still used in the United States, placed the freezing point of water at 32°F and the boiling point of water at 212°F.
- The <u>Celsius Scale</u>, previously known as the centigrade scale, is the SI unit for temperature. It is named after Anders Celsius and places the freezing point of water at 0°C and the boiling point of water at 100°C. This scale is used almost everywhere else in the world.

 $T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32]$ $T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32$ $\Delta T(^{\circ}F) = \frac{9}{5}\Delta T(^{\circ}C)$

- The temperature of an object is directly related to the kinetic energy of the random motions of its constituent particles. The minimum possible temperature would be the temperature associated with no motion at all, zero kinetic energy. This temperature is called <u>Absolute</u> <u>Zero</u>.
- The <u>Kelvin Scale</u> is the same scale as the Celsius Scale with zero moved from the freezing point of water to absolute zero, where motion stops.

$$0 K = -273.15^{\circ}C$$
 $T(^{\circ}C) = T(K) - 273.15$

Example: The high one day this summer was 104°F. What temperature is this in A) Celsius and B) Kelvin?

$$T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32] = \frac{5}{9}[104 - 32] = \frac{5}{9}[72] = 40^{\circ}C$$

$$T(K) = T(^{\circ}C) + 273.15 = 40 + 273.15 = 313 K$$

Thermal Expansion

• In most cases, the atomic separation increases in solids as the temperature (internal energy) increases. This causes solids to expand (<u>Thermal Expansion</u>).



Each atom vibrates in a potential well created by its neighbors. Notice that the average separation (the center of the vibration) increases with energy due to the shape of the well.

$$L = L_0 [1 + \alpha \Delta T] \qquad \Delta L = \alpha L_0 \Delta T$$

• <u>The Coefficient of Thermal Expansion</u> (α) is a constant, but its value differs from material to material. The units of α are (°C)⁻¹ (one over a degree Celsius).

Example: An overpass is to being constructed from 20.0 m long slabs of concrete. The coefficient of thermal expansion for concrete is $1.10 \times 10^{-5} (^{\circ}C)^{-1}$. Determine the width of the gap needed between slabs at 78.0°F such that the concrete slabs won't touch the next slab until it reaches the maximum expected temperature of 132.0°F.

$$\Delta T (^{\circ}\text{F}) = 132.0^{\circ}\text{F} - 78.0^{\circ}\text{F} = 54.0^{\circ}\text{F} \qquad \Delta T (^{\circ}\text{C}) = \frac{5}{9}\Delta T (^{\circ}\text{F}) = \frac{5}{9}(54.0) = 30.0^{\circ}\text{C}$$
$$\Delta L = \alpha L_0 \Delta T = [1.10 \times 10^{-5} (^{\circ}\text{C})^{-1}](20.0 \text{ }m)(30.0^{\circ}\text{C}) = 6.60 \text{ }mm$$

<u>Heat and Temperature Change</u> $Q = cm\Delta T$

- All objects have **Internal Energy** (U) consisting of the random atomic and molecular motions.
- This energy can be transferred from one object to another, and the movement of this energy is called <u>Heat</u> (Q).
- Heat (Q) flows from higher temperature to lower temperature.

$$Q = cm\Delta T$$

- Q is the heat added (in J).
- c is the specific heat capacity of that substance (units $\frac{J}{ka^{\circ}}$)
- m is the mass of the object (in kg)
- ΔT is the change in temperature (in °C or K)
- In gases the volume (V), the pressure (P), and the temperature (T) are dependent. Changing the temperature must change either the volume of the pressure.
- The specific heat capacity of a gas is different depending upon whether the volume or pressure is held constant (cV or cP).

Example: A tub can hold up to 404 kg of water. It has two taps. One tap produces cool water at 15.0° C, and the other produces hot water at 45.0° C. How much cool water should you add in order to have a full tub of warm water at 37.0° C?

The heat lost by the hot water must raise the temperature of the cold water.

$$Q_C + Q_H = 0$$
 $cm_c \Delta T_c + cm_H \Delta T_H = 0$ $m_c \Delta T_c + m_H \Delta T_H = 0$

$$m_c(T_f - T_C) + m_H(T_f - T_H) = 0$$
 $m_c(37.0^{\circ}\text{C} - 15.0^{\circ}\text{C}) + m_H(37.0^{\circ}\text{C} - 45.0^{\circ}\text{C}) = 0$

$$m_c(22.0^{\circ}\text{C}) - m_H(8.0^{\circ}\text{C}) = 0$$
 $m_c(22.0^{\circ}\text{C}) - (m_{Tot} - m_c)(8.0^{\circ}\text{C}) = 0$

 $m_c(22.0^{\circ}\text{C}) - m_{Tot}(8.0^{\circ}\text{C}) + m_c(8.0^{\circ}\text{C}) = 0$ $m_c(30.0^{\circ}\text{C}) = m_{Tot}(8.0^{\circ}\text{C})$

$$m_c = m_{Tot} \frac{(8.0^{\circ}\text{C})}{(30.0^{\circ}\text{C})} = (404 \text{ kg}) \frac{(8.0^{\circ}\text{C})}{(30.0^{\circ}\text{C})} = 108 \text{ kg}$$

Calories and James Joule

- Initially it wasn't known that heat and mechanical energy were equivalent. Consequently, heat had its own units (calorie).
- A <u>calorie</u> is defined as the amount of heat needed to raise the temperature of one gram of water by 1°C.
- In 1847, James Joule performed a definitive experiment showing that heat and mechanical energy were equivalent.



- Falling masses were used to turn agitators inside an insulated fluid.
- The potential energy of the masses was converted into a rise in temperature of the fluid.

 $1 \, cal = 4.186 \, J$

Nutritionist use "Calories" to measure the energy value of food, but these are actually kilo-calories. "cal" = calorie. "Cal" = kcal = kilo-calorie.

Heat and Phase Change

• Traditionally there are 3 states ("phases") of matter: solids, liquids, and gases

Today some would include plasmas, Bose-Einstein condensates, quark-gluon plasmas, etc.



• When a substance changes from one phase to another an amount of heat must be added (or removed) for the atoms/molecules rearrange themselves.

$$Q = mL$$

- 'Q' is the heat added to the substance, and 'm' is the mass of the substance.
- The Latent Heat (L) is a property of the material and depends upon which transition is occurring.
 - Solid/liquid transitions use the Latent Heat of Fusion (L_F)
 - Liquid/gas transitions use the Latent Heat of Vaporization (L_v)
 - Solid/gas transitions use the Latent Heat of Sublimation (Ls)

Example: 3.78 kg of water (a gallon) has been frozen into ice. How much heat is required to convert it from ice at -15.0°C into steam at 130°C? The latent heat of fusion for water/ice is 334 J/g. The latent heat of vaporization for water/steam is 2230 J/g. The specific heat of water is 4187 J/(kg·K). The specific heat for ice is 2108 J/(kg·K). The specific heat for steam is 1996 J/(kg·K).

Ice from -15°C to 0°C:
$$Q_1 = cm\Delta T = \left(2108\frac{J}{kg\cdot K}\right)(3.78 kg)(15.0°C) = 119,523.6 J$$

Ice to water at 0°C: $Q_2 = mL_F = \left(334\frac{J}{g}\right)(3780 g) = 1,262,520 J$
Water from 0°C to 100°C: $Q_3 = cm\Delta T = \left(4187\frac{J}{kg\cdot K}\right)(3.78 kg)(100°C) = 1,582,686 J$
Water to steam at 100°C: $Q_4 = mL_V = \left(2230\frac{J}{g}\right)(3780 g) = 8,429,400 J$
Steam from 100°C to 130°C: $Q_5 = cm\Delta T = \left(1996\frac{J}{kg\cdot K}\right)(3.78 kg)(30°C) = 226,346.4 J$
 $Q_{Tot} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$
 $Q_{Tot} = 119,523.6 J + 1,262,520 J + 1,582,686 J + 8,429,400 J + 226,346.4 J$
 $Q_{Tot} = 11.6 MJ$



As heat is added the temperature of the ice rises until it hits 0°C. Once the ice hits 0°C (the melting point of ice) additional energy only changes the state. Once it has been converted to a liquid (water) the temperature rises with added heat. At 100°C (the boiling point), additional energy changes the state into a gas (steam). Once the water has all been vaporized, additional heat can once again raise the temperature.

Heat Transfer

- There are 3 methods in which heat is transferred: convection, conduction, and radiation.
- In <u>Convection</u>, heat is transferred by the bulk movement of a gas or liquid.

For example, the hot air coming out of a blow dryer carries heat with it.



Water expands as it is heated, becoming less dense. This causes it to rise to the surface.

The rising water pushes warmer water from the center to the outsides, cooling as it moves.

Cooler water on the outer edges falls replacing the water that has risen in the center.

The flow of liquid or gas responsible for transporting heat is called a **<u>Convection</u>** <u>**Current**</u>.

• In <u>Conduction</u>, heat is transferred directly through a material without bulk movement of that material.

For example, if you were to grab the handle of a hot iron skillet, heat would flow through the metal and into your hand.

• Thermal Conductors readily transport heat.

Iron skillets are thermal conductors.

• Thermal insulators transport heat poorly.

Oven mitts are thermal insulators.



- *Q* is the heat transferred.
- *k* is the thermal conductivity of the conducting material.
- *A* is the cross-sectional area of the conducting material (the area through which heat can flow).
- ΔT is the temperature difference between the ends. $\Delta T = T_H - T_C$
- *L* is the length of the thermal conductor (how far the heat must travel).
- *t* is the time interval.
- *P* is the conductive power.

Example: The temperature of an oven is $375^{\circ}F$ when baking bread. The temperature of the kitchen outside the oven door is $78.0^{\circ}F$. The surface of the oven door is 0.300 m^2 and is coated with a 2.00 mm thick calcium silicate insulator with a thermal conductivity of $0.0500 \text{ J/(s} \cdot \text{m} \cdot ^{\circ}C)$. (A) How much energy is used to operate the pre-heated oven for 5.00 hours if losses through the other surfaces are negligible compared to losses through the door, and (B) How much does this cost at 12ϕ per kilo-Watt-hour for energy?

$$\Delta T = T_H - T_C = 375^{\circ}\text{F} - 78.0^{\circ}\text{F} = 297^{\circ}\text{F} \qquad \Delta T(^{\circ}\text{C}) = \frac{5}{9}\Delta T(^{\circ}\text{F}) = \frac{5}{9}(297^{\circ}\text{F}) = 165^{\circ}\text{C}$$

$$5.00 \ hours\left(\frac{3600 \ s}{1 \ hour}\right) = 18000 \ s$$

$$Q = \left(\frac{kA\Delta T}{L}\right)t = \left(\frac{\left(0.0500 \ \frac{J}{s \cdot m \cdot ^{\circ}\text{C}}\right)(0.300 \ \text{m}^2)(165^{\circ}\text{C})}{2.00 \times 10^{-3} \ m}\right)(18000 \ s) = 2.2275 \times 10^7 \ J$$

$$2.2275 \times 10^7 J\left(\frac{\$ \ 0.12}{1 \ kW \cdot hr}\right) \left(\frac{1 \ kW \cdot hr}{3.60 \times 10^6 J}\right) = \$ \ 0.7425$$

- In <u>Radiation</u>, heat is transferred by electromagnetic waves (typically in the infra-red range).
 - Objects appear black because they absorb most of the light and reflect little. Objects that readily absorb radiation are also good emitters.
 - A <u>Black</u> <u>Body</u> is an ideal emitter of radiation.

• Stefan-Boltzmann Law of Radiation
$$Q = e\sigma T^4 At$$
 $P = \frac{Q}{t} = e\sigma T^4 A$

- Q is the heat transferred.
- e is the emissivity (0 to 1), a measure of how close the object is to an ideal black body (e = 1)
- Stefan-Boltzmann constant (σ): $\sigma = 5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}$
- T is the temperature in Kelvin.
- A is the surface area of the object (the area emitting the EM waves).
- t is the time interval.
- P is the radiant power.

Example: Betelgeuse is a red supergiant star in the constellation Orion. It has a surface temperature of 3590 K and emits a radiant power of 4.64×10^{31} W. Determine the radius of Betelgeuse assuming it is approximately spherical and a perfect emitter (e = 1).

$$P = \frac{Q}{t} = e\sigma T^4 A = \sigma T^4 A = \sigma T^4 (4\pi r^2) = 4\pi\sigma T^4 r^2$$
$$r^2 = \frac{P}{4\pi\sigma T^4}$$
$$r = \sqrt{\frac{P}{4\pi\sigma T^4}} = \sqrt{\frac{4.64 \times 10^{31} \text{ W}}{4\pi \left(5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}\right)(3590 \text{ K})^4}} = 6.26 \times 10^{11} m$$

For comparison, the average separation between the sun and Jupiter is 7.78×10^{11} m.

The Atomic Mass Scale

• The <u>Atomic Mass Unit</u> (u) is the mass of a carbon-12 atom (¹²C) divided by 12. The atomic mass unit is roughly the mass of a proton or neutron.

$$u = 1.66 \times 10^{-27} \, kg$$

• The mass of an atom in atomic mass units is the same as the number of nucleons in the atom.

⁴He has 4 nucleons (2 protons and 2 neutrons). The mass of a ⁴He atom is 4u.

- The mass of a molecule is the sum of the masses of its atoms.
- One <u>Mole</u> (mol) of a substance contains Avogadro's number (6.022×10^{23}) of particles (could be atoms or molecules), which is the number of atoms in 12.0 g of carbon-12.

$$N_A = 6.022 \times 10^{23}$$

This creates a correspondence between the masses of atoms (atomic mass scale) and masses at the macroscopic scale (the masses of moles in grams).

For example, the mass of 1 mol of a substance with atomic mass 28 u (silicon) is going to be 28 g.

 $N = \# of \ particles \ in \ a \ substance$ $n = \# of \ moles \ of \ a \ substance$ $n = \frac{N}{N_A}$

 $\underline{Ideal \ Gas \ Law} \qquad PV = nRT \qquad PV = NkT$

- The ideal gas law can be written in two ways. One uses 'n', the number of moles, and the other uses 'N' the number of particles.
- P is the pressure of the gas. R is the <u>Universal Gas Constant</u>: $R = 8.314 \frac{J}{mol/K}$
- V is the container volume. K is the <u>Boltzmann</u> Constant: $k = 1.38 \times 10^{-23} \frac{J}{V}$
- T is the gas temperature. For the two equations to be equal, $R = N_A k$ nR = Nk

The units of both sides of the ideal gas law are Joules. This is an energy equation!

Example: The Airlander 10 is the largest operational aircraft in the world (as of 2016). It typically contains 38,000 cubic meters of helium at 1.01 atm. If the temperature is 20.0°C, how many moles of helium are needed to fill the craft?

$$P = (1.01 \text{ atm}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}}\right) = 102,313 \text{ Pa} \qquad T(K) = 20.0^{\circ}\text{C} + 273.15 = 293.15 \text{ K}$$
$$PV = nRT \qquad n = \frac{PV}{RT} = \frac{(102,313 \text{ Pa})(38000 \text{ m}^3)}{(R=8.314 \frac{J}{mol\cdot K})(293.15 \text{ K})} = 1.60 \times 10^6 \text{ moles}$$

- Using the Ideal Gas Law
 - Absolute pressure must be used in the ideal gas law (not gauge pressure). Normally this must also be in Pascal (Pa) except as noted below.
 - Temperature must be in Kelvin.

- Normally the volume must be in cubic meters (m³) except as noted below.
- Often we are comparing two different states of the same gas at two different time periods. If so, we can use that PV/nT (or PV/NT) is a constant.

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \quad ... \text{Or if n is constant:} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

- When comparing two different states of the same gas, multiplicative conversion factors will cancel. This allows most units to be used for volume (liters, cm³, etc) and pressure (atm).
- Conversions that involve adding/subtracting cannot be used. Temperature must be in Kelvin, and absolute pressure (not gauge pressure) must be used.
- If the temperature is held constant, you get <u>**Boyle's Law**</u>: $P_1V_1 = P_2V_2$

• If the pressure is held constant, you get Charles' Law:
$$\frac{v_1}{T_1} = \frac{v_2}{T_2}$$

• If the volume is held constant, you get <u>Gay-Lussac's Law</u>: $T_1 = \frac{T_2}{T_1}$

Example: A driver in Texas carefully prepares her car for a trip. This includes making sure the tires are inflated to a gauge pressure of 30.0 lbs per square inch (207 kPa). The tires have a volume of 1.64×10^{-2} m³ and were filled at 95.0°F. Three days later the driver is in Canada at a temperature of 23.0°F. Assuming no air was lost from the tires since they were filled, what is the gauge pressure of the tires in Canada?

$$P_1 = P_{Absolute} = P_{Gauge} + P_{atm} = 207 \, kPa + 101.3 \, kPa = 308.3 \, kPa$$

$$T_1(^{\circ}\text{C}) = \frac{5}{9}[T_1(^{\circ}\text{F}) - 32] = \frac{5}{9}[95.0^{\circ}\text{F} - 32] = 35.0^{\circ}\text{C} \qquad T_1(K) = 35.0^{\circ}\text{C} + 273.15 = 308.15 \ K$$
$$T_2(^{\circ}\text{C}) = \frac{5}{9}[T_2(^{\circ}\text{F}) - 32] = \frac{5}{9}[23.0^{\circ}\text{F} - 32] = -5.0^{\circ}\text{C} \qquad T_2(K) = -5.0^{\circ}\text{C} + 273.15 = 268.15 \ K$$

As no air was lost from the tires, n is constant. The volume of the tires is given. We can assume that is fixed. So V is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad P_2 = \frac{T_2}{T_1} P_1 = \frac{268.15 \, k}{308.15 \, k} (308.3 \, kPa) = 268.3 \, kPa$$

 $P_{Gauge} = P_{Absolute} - P_{atm} = 268.3 kPa - 101.3 kPa = 167.0 kPa$ (roughly 24.2 lbs/in²)

• <u>Standard Temperature and Pressure (STP)</u>

- Standard Temperature is 0°C (or 273.15 K)
- Standard Pressure is 1 atm $(1.013 \times 10^5 \text{ Pa})$
- We can determine the volume of 1 mole of an ideal gas at STP.

$$V = \frac{nRT}{P} = \frac{(1 \text{ mol})\left(8.314 \frac{J}{\text{ mol} \cdot K}\right)(273.15 \text{ K})}{(1.013 \times 10^5 \text{ Pa})} = 0.02242 \text{ m} (22.4 \text{ liters})$$

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The Kinetic Theory of Gas

Our goal is to find a relationship between the microscopic properties of the particles that make up a gas and the macroscopic properties of the gas.

Let's begin with a cube with sides of length L and the simplest possible gas inside it, a single particle moving with velocity v.



The pressure is the force per area:
$$P = \frac{F}{A}$$

The force is the momentum per time: $F = \frac{\Delta P}{\Delta t}$

The time interval is the time it takes the particle to return:

$$v\Delta t = 2L$$
 $\Delta t = \frac{2L}{v}$

The change in momentum of the wall is equal and opposite to the change in momentum of the particle.

$$\Delta P_{wall} = -(\Delta P_{particle}) = -(P_{final} - P_{initial}) = -[m(-\nu) - m\nu] = 2m\nu$$

$$F = \frac{\Delta P}{\Delta t} = \frac{2m\nu}{\frac{2L}{\mu}} = \frac{m\nu^2}{L} \qquad P = \frac{m\nu^2}{\frac{L}{L^2}} = \frac{m\nu^2}{L^3}$$

This is the pressure created by a single particle. Now let's fill the container with N particles, but we must account for the "equipartition theorem".

Equipartition of Energy: Each degree of freedom (dimension) must have the same amount of energy carried through it.

To account for this, we will have all of our particles moving in one-dimension with a third moving in each direction. Then we will calculate the force and then the pressure on one face.

This may seem rather contrived, but if left for a short time collisions will return this system to a completely random state. Consequently, these states are equivalent (at least for our purposes here).

Instead of v^2 , which varies from particles to particle, we must use the average value of v^2 . To do this we will use "rms" values (which stands for "root mean square", the square root of the mean of squares).

$$v_{rms} = \sqrt{\overline{v^2}} \qquad \overline{v^2} = v_{rms}^2$$

$$F_{Tot} = (\# of \ particles)(F_{avg}) = \left(\frac{N}{3}\right) \left(\frac{m\overline{v^2}}{L}\right) = \frac{Nmv_{rms}^2}{3L}$$

$$P = \frac{F_{Tot}}{A} = \frac{\frac{Nmv_{rms}^2}{3L}}{\frac{12}{L^2}} = \frac{Nmv_{rms}^2}{3L^3} = \frac{Nmv_{rms}^2}{3V} \qquad PV = \frac{1}{3}Nmv_{rms}^2 = NkT$$

$$mv_{rms}^2 = 3kT \qquad KE_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$
Internal Energy of a Monoatomic Ideal Gas: $U = N\left(\frac{3}{2}kT\right) = \frac{3}{2}NkT = \frac{3}{2}nRT$

Example: Determine the change in internal energy of helium gas in a 56.0 m^3 tank as it is heated to 35.0°C starting in standard temperature and pressure.

$$V = 56.0 m^{3} \qquad T_{Final} = 35^{\circ}\text{C} + 273.15 = 308.15 K \qquad T_{init} = 273.15 K \qquad P_{init} = 101.3 kPa$$
$$\Delta U = U_{final} - U_{init} = \frac{3}{2}nRT_{final} - \frac{3}{2}nRT_{init} = \frac{3}{2}nR(T_{final} - T_{init})$$

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$$PV = nRT$$
 $nR = \frac{PV}{T} = \frac{P_{init}V_{init}}{T_{init}}$

$$\Delta U = \frac{3}{2} \left(\frac{P_{init} V_{init}}{T_{init}} \right) \left(T_{final} - T_{init} \right) = \frac{3}{2} \left(\frac{(1.013 \times 10^5 \ Pa)(56.0 \ m^3)}{273.15 \ K} \right) (308.15 \ K - 273.15 \ K) = 1.09 \ MJ$$

Thermodynamics

- <u>Thermodynamics</u> is the relationship between heat and work.
- The **System** is a collection of objects of interest.
- The <u>Surroundings</u> are everything else.
- Typically the system is surrounded by walls.
 - **<u>Diathermal</u>** walls permit heat flow between the system and surroundings.
 - <u>Adiabatic</u> walls do not permit heat flow.
- The <u>State of the System</u> refers to the physical condition of the system (pressure, temperature, etc.)
- Two systems are in <u>Thermal Equilibrium</u> if there is no heat flow when the two systems are in thermal contact (i.e. heat could flow).

<u>The 0th Law of Thermodynamics</u>: Two systems that are individually in thermal equilibrium with a third system are in thermal equilibrium with each other.

Consider the equation for heat flow via conduction. In thermal equilibrium there is no heat flow (Q = 0). What condition makes this so?



L, *k*, and *A* are constants, and *Q* must be zero for all values of *t*. Two objects are in thermal equilibrium if they are at the same temperature.

Two systems that are individually at the same temperature as a third system are at the same temperature as each other.

The 1st Law of Thermodynamics

The 1st law of thermodynamics is applying conservation of energy with heat.

$$\Delta U = U_{final} - U_{init} = Q - W$$

The change in the internal energy of a system (ΔU) is equal the heat ADDED TO the system (Q) minus any work done BY the system (W).

The internal energy of a system depends only upon the state of a system and not on the method by which the system arrives at that state. In other words, the internal energy of a system is solely a function of the state of that system.

Isobaric Process (Constant Pressure)



The force from the gas pressure at the bottom of the piston must support the weight of the mass and the piston. As that weight is fixed (constant), the pressure must also be fixed (constant). If we account for outside air pressure (also fixed) it doesn't change that the pressure is constant.

When heat (energy) is added the temperature rises. The ideal gas law(PV = nRT) tells us that the temperature can't rise without a change in volume and/or pressure. As the pressure is fixed, the gas expands

W (work done **BY** the gas) = $Fs = (PA)s = P(As) = P \cdot \Delta V = P \cdot (V_{final} - V_{init})$



In a P-V Diagram, where the pressure (P) is plotted against volume (V), the work (W) is the area under the curve ($W = P \cdot \Delta V$).

Example: A gas is held at a constant pressure of 150 kPa as it expands, doing 750 J of work in the process. The initial volume of the gas is 2.00×10^{-3} m³. Determine (A) the final volume of the gas, and (B) the change in internal energy of the gas.

$$W = P \cdot \Delta V = P \cdot \left(V_{final} - V_{init} \right) \qquad V_{final} - V_{init} = \frac{W}{P}$$

$$V_{final} = \frac{W}{P} + V_{init} = \frac{750 J}{1.50 \times 10^5 Pa} + 2.00 \times 10^{-3} m^3 = 7.00 \times 10^{-3} m^3$$

$$\Delta U = U_{final} - U_{init} = \frac{3}{2} nRT_{final} - \frac{3}{2} nRT_{init} = \frac{3}{2} PV_{final} - \frac{3}{2} PV_{init} = \frac{3}{2} P(V_{final} - V_{init}) = \frac{3}{2} P \cdot \Delta V = \frac{3}{2} W$$
$$\Delta U = \frac{3}{2} W = \frac{3}{2} (750 J) = 1125 J$$

Isochoric Process (Constant Volume)

- If the volume is constant no work is done: $W = F \cdot d = F \cdot 0 = 0$
- Any heat added changes the internal energy of the gas: $\Delta U = Q W = Q$

Adding heat increases the internal energy, which also increases the temperature of the gas. Increasing the temperature of the gas must increase the pressure (as the volume is constant).

$$Q = \Delta U = \frac{3}{2}Nk\Delta T$$
 $P = \frac{NkT}{V} = \frac{Nk}{V}(T_0 + \Delta T)$

Isothermal Process (Constant Temperature)



Heat entering the system is immediately converted to work via expansion.

Example: 5.00 moles of a monoatomic gas is held at a constant temperature as it is compressed from 5.00 liters to 2.50 liters. 8.95 kJ of work is done on the gas to compress it. Determine (A) the change in energy of the gas, (B) the heat added to the gas, and (C) the temperature of the gas.

- A) $\Delta U = Q W = 0$
- B) $Q = W = -8.95 \, kJ$

It must be negative. Work was done "to the gas", not "by the gas".

C)
$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$
 $T = \frac{W}{nR \ln\left(\frac{V_2}{V_1}\right)} = \frac{-8950 J}{(5 mol)\left(8.314 \frac{J}{mol \cdot K}\right) \ln\left(\frac{2.50 l}{5.00 l}\right)} = 311 K$

<u>Adiabatic Process</u> (Q = 0) No heat is added or removed

$$\Delta U = Q - W = -W \qquad W = -\Delta U = -\frac{3}{2}Nk\Delta T = -\frac{3}{2}Nk(T_{final} - T_{init}) = \frac{3}{2}Nk(T_{init} - T_{final})$$

Example: 10.0 moles of a monoatomic gas expands adiabatically. Its temperature falls from 425 K to 300 K. Determine (A) the work done by the gas, and (B) the change in the internal energy.

$$W = \frac{3}{2}nR(T_{init} - T_{final}) = \frac{3}{2}(10.0 \text{ mol})\left(8.314\frac{J}{mol\cdot K}\right)[425 \text{ K} - 300 \text{ K}] = 15.6 \text{ kJ}$$
$$\Delta U = -W = -15.6 \text{ kJ}$$